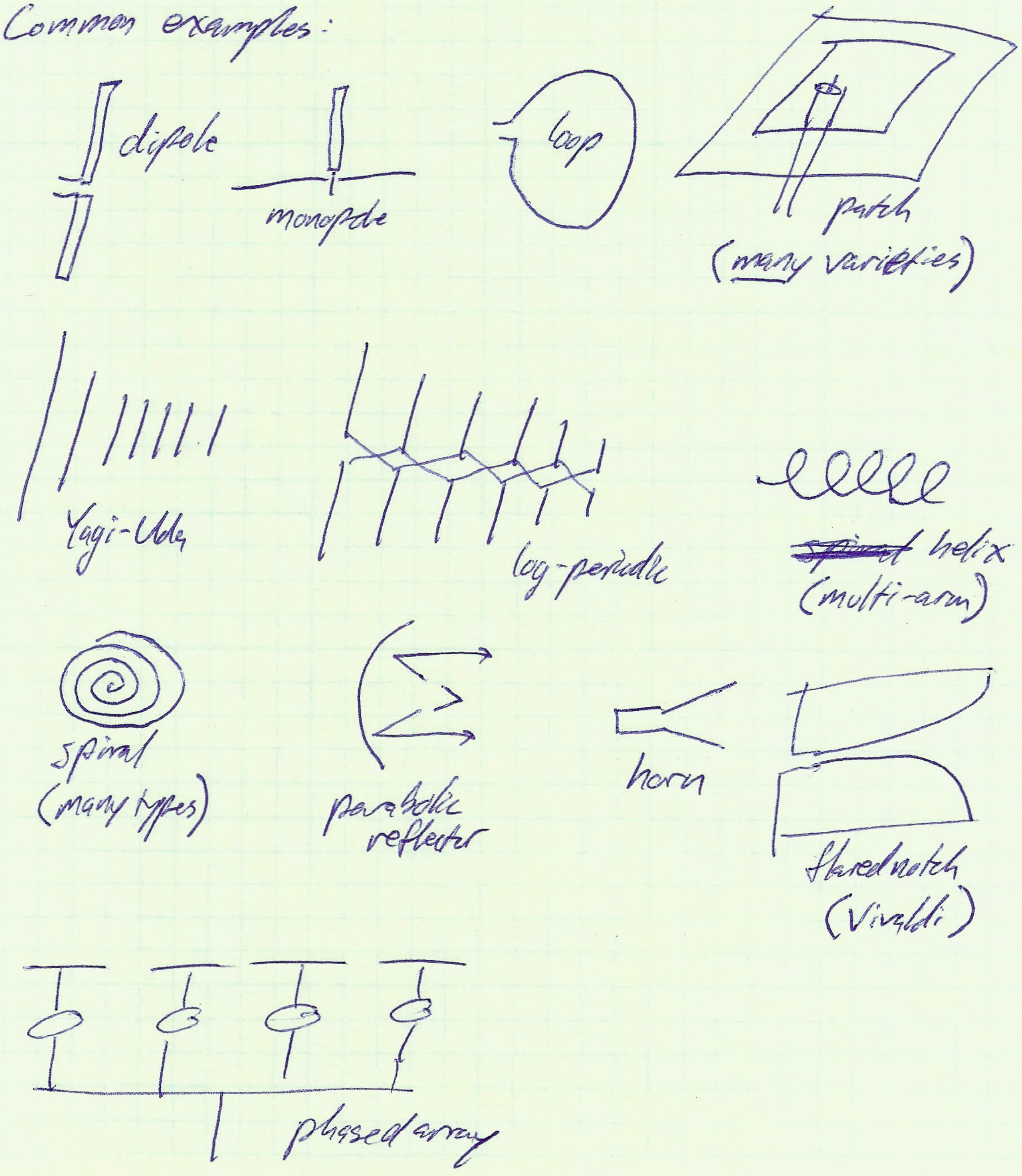
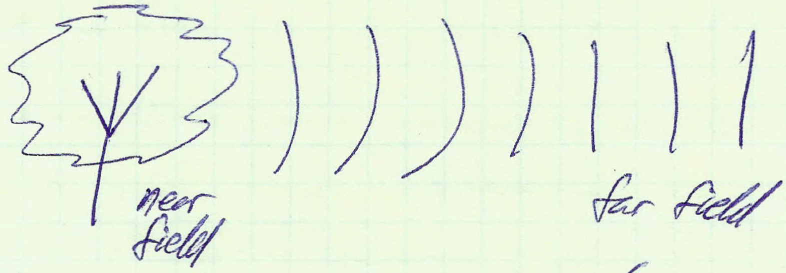
ECE107 | Electromagnetism | Lecture 23 p1

Antennas

- transform a wave in a waveguide to a wave in free space
- reciprocal device : works for transmit, the same as receive
- have a radiation pattern
  - → gain with respect to ideal isotropic radiator
  - → EIRP : effective isotropic radiated power $= P \times G$
- have a polarization - linear, circular, elliptical in general
- have an input impedance - typically match to 50Ω

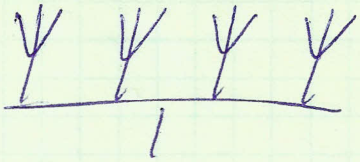
Common examples:





(contains field terms that decay faster than R^{-1})

(approximates plane waves, phase front is nearly flat)



Antenna arrays:
pattern = element pattern × array factor

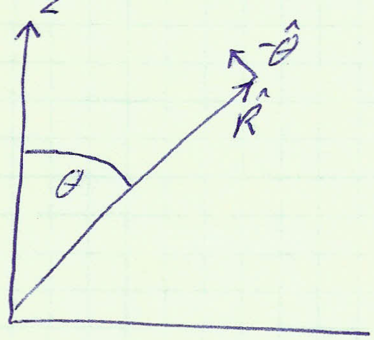
• The Hertzian Dipole (thin wire, $l < \lambda/50$)

$\uparrow \quad i(t) = I_0 \cos \omega t = \mathrm{Re}\left[I_0 e^{j\omega t} \right]$

$$\tilde{A}(\vec{R}) = \frac{\mu_0}{4\pi} \int_{v'} \frac{\tilde{J} e^{-jkR'}}{R'} dv' \quad \text{for } R' \gg l, \; R' \approx R$$

$$\tilde{A} = \frac{\mu_0}{4\pi} \frac{e^{-jkR}}{R} \int_{-l/2}^{l/2} \hat{z} I_0 dz = \hat{z} \frac{\mu_0}{4\pi} I_0 l \underbrace{\left(\frac{e^{-jkR}}{R} \right)}_{\text{spherical propagation factor}}$$

R = range
θ : zenith angle (more commonly elevation angle)
ϕ : azimuth angle



$$\hat{z} = \hat{R} \cos\theta - \hat{\theta} \sin\theta$$

$$\tilde{A} = (\hat{R}\cos\theta - \hat{\theta}\sin\theta) \frac{\mu_0 I_0 l}{4\pi} \left(\frac{e^{-jkR}}{R} \right)$$

$$\tilde{A} = \hat{R}\tilde{A}_R + \hat{\theta}\tilde{A}_\theta + \hat{\phi}\tilde{A}_\phi$$

$$\tilde{A}_R = \frac{\mu_0 I_0 l}{4\pi} \cos\theta \left(\frac{e^{-jkR}}{R} \right), \quad \tilde{A}_\theta = \frac{-\mu_0 I_0 l}{4\pi} \sin\theta \left(\frac{e^{-jkR}}{R} \right), \quad \tilde{A}_\phi = 0$$

$$\tilde{H} = \frac{1}{\mu_0} \nabla \times \tilde{A} = \frac{1}{\mu_0} \begin{vmatrix} \frac{\hat{R}}{R^2 \sin\theta} & \frac{\hat{\theta}}{R\sin\theta} & \frac{\hat{\phi}}{R} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \tilde{A}_R & R\tilde{A}_\theta & R\sin\theta A_\phi \end{vmatrix}$$

H will only be in ϕ direction, lets only solve that one

$$\tilde{H}_\phi = \frac{-I_0 l}{4\pi} \sin\theta (-jk) e^{-jkR} - \frac{I_0 l}{4\pi} (-\sin\theta) \frac{e^{-jkR}}{R}$$

$$= \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin\theta$$

$$\tilde{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \tilde{H}$$

$$\tilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos\theta$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin\theta$$

Far field approximation $\rightarrow kR = 2\pi R/\lambda \gg 1$ or $R \gg \lambda$

terms in $\frac{1}{(kR)^2}$ and $\frac{1}{(kR)^3} \rightarrow 0$

$$\tilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \sin\theta$$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0}$$

Power Density

$$\vec{S}_{av} = \frac{1}{2} Re\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right]$$

$$= \hat{R}\left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}\right)\sin^2\theta$$

Normalized Radiation Intensity

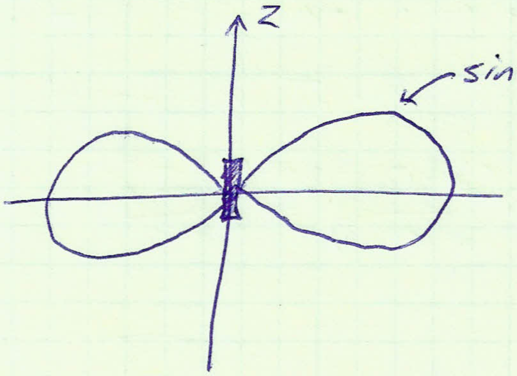
$$F(\theta,\phi) = \frac{S(R,\theta,\phi)}{S_{max}(R)}$$

$$S_{max} = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} \approx \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

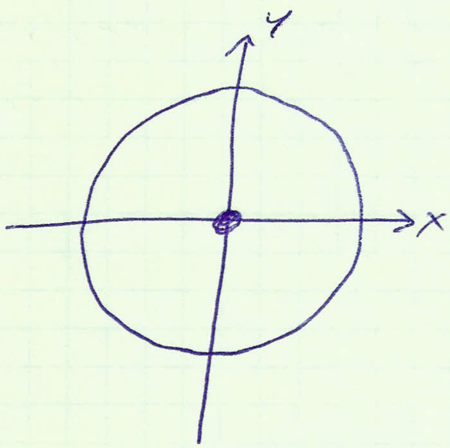
using $k = \frac{2\pi}{\lambda}$, $\eta_0 \approx 120\pi$
$(= 377)$

Proportional to I_0^2 and $(l/\lambda)^2$, $\frac{1}{R^2}$

$$F(\theta,\phi) = \sin^2\theta$$



elevation pattern



azimuth pattern